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# Study of $B_c^- \to X(3872)\pi^-(K^-)$ decays in the covariant light-front approach

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**Abstract.** In the covariant light-front quark model, we calculate the form factors of $B_c^- \to J/\psi$ and $B_c^- \to X(3872)$. Since the factorization of the exclusive processes $B_c^- \to J/\psi\pi^-(K^-)$ and $B_c^- \to X(3872)\pi^-(K^-)$ can be proved in the soft-collinear effective theory, we can easily get the branching ratios for these decays from the form factors. Taking the uncertainties into account, our results for the branching ratio of $B_c^- \to J/\psi\pi^-(K^-)$ are consistent with previous studies. By identifying $X(3872)$ as a $1^{++}$ charmonium state, we obtain $\mathrm{BR}(B_c^- \to X(3872)\pi^-) = (1.7^{+0.7+0.1+0.4}_{-0.6-0.2-0.4}) \times 10^{-4}$ and $\mathrm{BR}(B_c^- \to X(3872)K^-) = (1.3^{+0.5+0.1+0.3}_{-0.5-0.2-0.3}) \times 10^{-5}$. Assuming $X(3872)$ to be a $1^{--}$ state, the branching ratios will be one order of magnitude larger than those of the $1^{++}$ state. These results can easily be used to test the charmonium description for this mysterious meson $X(3872)$ at the LHCb experiment.

## 1 Introduction

$X(3872)$ was first observed by Belle in the exclusive decay $B^\pm \to K^\pm X \to K^\pm\pi^+\pi^- J/\psi$ [1], and subsequently confirmed by the CDF, D0 and BaBar collaborations in various decay and production channels [2–4]. At present a definite answer to the question of its internal properties is not well established, but the current experimental data strongly support a $1^{++}$ state [5]. Enormous interest in the study of $\bar{c}c$ resonance spectroscopy followed this discovery and there exist many interpretations of this meson. The first and most natural assignment of this state is the first radial excitation of the $1P$ charmonium state $\chi_{c1}$ [6]. However, this interpretation has encountered two difficulties: its decay width ($< 2.3$ MeV, 95% C.L.) is tiny compared with other charmonia; and there is a gap of about 100 MeV between the measured mass and the quark model prediction [7]. Motivated by these two difficulties, many non-charmonium explanations were proposed, such as it being a $\bar{c}cg$ hybrid meson [8], a glueball [9], a diquark cluster [10, 11], and a molecular state [12–15]. But in fact, there are few experimental data that could provide a clear discrimination among these descriptions, and this makes the situation more obscure. Recently the CLQCD collaboration studied the mass for the first excited states of $1^{++}$ charmonium and found that it is consistent with the measured mass of $X(3872)$ [16]. Consistence indicates that $X(3872)$ can be the first radial excited state of $\chi_{c1}$, and it seems that the mass difficulty trails off. Now, in order to investigate the structure of this meson more clearly, a large amount of experimental data and theoretical studies on the productions and decays of $X(3872)$ are strongly needed.

In $B_{u,d,s}$ decays involving the charmonium final sates, the emitted meson is a heavy charmonium. The non-factorizable contribution should be large to induce large uncertainties [17]. As the energy release is limited, these decays may also be polluted by the final state interactions, which are non-perturbative in nature. But fortunately the production of charmonia in $B_c$ decays could provide unique insight in these mesons. Since the emitted meson here is a light meson ($\pi$ or $K$), the factorization of $B_c \to (\bar{c}c)M$ ($M$ is a light meson) could be proved in the framework of the soft-collinear effective theory (SCET) to all orders of the strong coupling constant in the heavy quark limit, which is similar to $\bar{B}^0 \to D^+\pi^-$ and $B^- \to D^0\pi^-$ [18]. The decay matrix element can be decomposed into a $B_c \to (\bar{c}c)$ form factor and a convolution of a short distance coefficient with the light-cone wave function of the emitted light meson.

Although SCET provides a powerful framework to study the factorization of the exclusive modes, the non-perturbative form factors could not be directly studied. We can only extract them via the experimental data or rely on some non-perturbative method. In the present paper, we will use the light-front quark model to calculate these $B_c \to M(\bar{c}c)$ form factors. As pointed out in [19], the light-front QCD approach has some unique features that are particularly suitable for use in a description of a hadronic bound state. The light-front quark model [20–23] can provide a relativistic treatment of the movement of the hadron and also give a full treatment of the hadron spin by using the so-called Melosh rotation. Light-front

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wave functions, which describe the hadron in terms of their fundamental quark and gluon degrees of freedom, are independent of the hadron momentum and thus are explicitly Lorentz invariant. Furthermore, in the covariant light-front approach [24], the spurious contribution, which is dependent on the orientation of the light front, is eliminated by including the zero-mode contributions properly. This covariant model has been successfully extended to the study of the decay constants and form factors of the ground state $s$-wave and the low-lying $p$-wave mesons [25–28].

The paper is organized as follows. The formalism for the form factor calculations, taking $B_c^- \to J/\psi$ as an example, is presented in the next section. The numerical results for form factors and decay rates of $B_c^- \to J/\psi\pi^-(K^-)$, $B_c^- \to J/\psi\rho^-(K^{*-})$, $B_c^- \to X(3872)\pi^-(K^-)$ and $B_c^- \to X(3872)\rho^-(K^{*-})$ are given in Sect. 3. The conclusion is given in Sect. 4.

## 2 Calculation of the form factors and the branching ratios

In the following, we use $X$ to denote $X(3872)$ for simplicity. Different from the $B_{u,d,s}$ mesons, the $B_c^-$ system consists of two heavy quarks, $b$ and $c$, which can decay individually. Here we will consider $b$ decays, while $\bar{c}$ acts as a spectator. At the quark level, $B_c^- \to J/\psi\pi^-$ and $B_c^- \to X(3872)\pi^-$ decays are characterized by the $b \to (cd\bar{u})$ transition and the corresponding effective Hamiltonian is given by [1]

$$\mathcal{H}_{\text{eff}} = \frac{G_{\text{F}}}{\sqrt{2}} V_{cb} V_{ud}^* [C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu)] + \text{h.c.}\,, \quad (1)$$

where $V_{ij}$ are the corresponding CKM matrix elements. The local four-quark operators $O_{1,2}$ are defined by

$$\begin{aligned} O_1(\mu) &= (\bar{c}_\alpha b_\beta)_{V-A} (\bar{d}_\beta u_\alpha)_{V-A}\,, \\ O_2(\mu) &= (\bar{c}_\alpha b_\alpha)_{V-A} (\bar{d}_\beta u_\beta)_{V-A}\,, \end{aligned} \quad (2)$$

where $\alpha$ and $\beta$ are the color indices. Since the four quarks in the operators are different from each other, there is no penguin contribution, and thus there is no $CP$ violation. The left handed current is defined as $(\bar{q}_\alpha q_\beta)_{V-A} = \bar{q}_\alpha \gamma_\nu (1-\gamma_5) q_\beta$. For the $b \to (cs\bar{u})$ transition, $V_{ud}^*$ is replaced by $V_{us}^*$, while the $d$ quark field in the four-quark operator is replaced by $s$. With the effective Hamiltonian given above, the matrix element for the $B_c^- \to J/\psi\pi^-$ transition can be expressed as

$$\begin{aligned} \mathcal{M} &= \langle J/\psi(P'', \varepsilon''^*)\pi | \mathcal{H}_{\text{eff}} | B_c^-(P') \rangle \\ &= \frac{G_{\text{F}}}{\sqrt{2}} V_{cb} V_{ud}^* a_1(\mu) \langle J/\psi(P'', \varepsilon''^*)\pi | O_2(\mu) | B_c^-(P') \rangle\,, \end{aligned} \quad (3)$$

[1] For a review, see [29].

with $P'^{(\prime\prime)}$ being the incoming (outgoing) momentum, $\varepsilon''^*$ the polarization vector of $J/\psi$ and $a_1 = C_2 + C_1/3$ the Wilson coefficient.

In the effective Hamiltonian, the degrees of freedom heavier than the $b$ quark mass $m_b$ scale are included in the Wilson coefficients, which can be calculated using perturbation theory. Then the task that is left is to calculate the operators' matrix elements between the $B_c^-$ meson state and the final states, which suffer large uncertainties. Nevertheless, the problem becomes tractable if factorization becomes applicable. Thanks to the development of SCET, the proof of the factorization can be accomplished in an elegant way [30, 31]. In SCET, the heavy meson is described by the heavy quarks $h_v$ and soft gluons $A_s$ in its rest frame; the final state light meson moves very fast, and it is described by the collinear quarks $\xi_c$ and the collinear gluons $A_c$. In [18], it has been shown that the collinear gluons do not connect to the particle in the heavy meson, while the soft gluons do not connect to those in the light meson to all orders in $\alpha_{\text{s}}$ and leading power in $\Lambda_{\text{QCD}}/m_{B_c}$. In phenomenological language, the non-factorizable diagrams cancel each other because of color transparency. Furthermore, there is no annihilation contribution as the quarks in the final state meson are different from each other. Thus the decay amplitude can be expressed as the product of the $B_c \to J/\psi$ form factor and a convolution of a short distance Wilson coefficient with the non-perturbative light-cone distribution amplitude of the light meson. Without the higher order QCD corrections, the convolution is reduced to the decay constant of the light meson.

The form factors for the $B_c \to J/\psi$ and $B_c \to X(3872)$ ($1^{++}$ state) transitions induced by the vector and axial-vector currents are defined by

$$\begin{aligned} &\langle J/\psi(P'', \varepsilon''^*) | V_\mu | B_c^-(P') \rangle \\ &= -\frac{1}{m_{B_c} + m_{J/\psi}} \epsilon_{\mu\nu\alpha\beta} \varepsilon''^{*\nu} P^\alpha q^\beta V^{PV}(q^2)\,, \end{aligned} \quad (4)$$

$$\begin{aligned} &\langle J/\psi(P'', \varepsilon''^*) | A_\mu | B_c^-(P') \rangle \\ &= \mathrm{i} \Big\{ (m_{B_c} + m_{J/\psi}) \varepsilon''^*_\mu A_1^{PV}(q^2) - \frac{\varepsilon''^* \cdot P}{m_{B_c} + m_{J/\psi}} P_\mu A_2^{PV}(q^2) \\ &\quad - 2m_{J/\psi} \frac{\varepsilon''^* \cdot P}{q^2} q_\mu \left[ A_3^{PV}(q^2) - A_0^{PV}(q^2) \right] \Big\}\,, \end{aligned} \quad (5)$$

$$\begin{aligned} &\langle X(P'', \varepsilon'') | V_\mu | B_c^-(P') \rangle \\ &= (m_{B_c} - m_X) \varepsilon^*_\mu V_1^{PA}(q^2) - \frac{\varepsilon^* \cdot P'}{m_{B_c} - m_X} P_\mu V_2^{PA}(q^2) \\ &\quad - 2m_X \frac{\varepsilon^* \cdot P'}{q^2} q_\mu \left[ V_3^{PA}(q^2) - V_0^{PA}(q^2) \right]\,, \end{aligned} \quad (6)$$

$$\begin{aligned} &\langle X(P'', \varepsilon'') | A_\mu | B_c^-(P') \rangle \\ &= -\frac{\mathrm{i}}{m_{B_c} - m_X} \epsilon_{\mu\nu\rho\sigma} \varepsilon^{*\nu} P^\rho q^\sigma A^{PA}(q^2)\,, \end{aligned} \quad (7)$$

where $P = P' + P''$, $q = P' - P''$, and the convention $\epsilon_{0123} = 1$ is adopted. To cancel the poles at $q^2 = 0$, we must have $A_3^{PV}(0) = A_0^{PV}(0)$, $V_3^{PA}(0) = V_0^{PA}(0)$. The form factor $A_3^{PV}(V_3^{PA})$ is related to the other form factors by

$$A_3^{PV}(q^2) = \frac{m_{B_c}+m_{J/\psi}}{2m_{J/\psi}} A_1^{PV}(q^2) - \frac{m_{B_c}-m_{J/\psi}}{2m_{J/\psi}} A_2^{PV}(q^2) ,$$
$$V_3^{PA}(q^2) = \frac{m_{B_c}-m_X}{2m_X} V_1^{PA}(q^2) - \frac{m_{B_c}+m_X}{2m_X} V_2^{PA}(q^2) . \tag{8}$$

Following the notation in [24–26], we use the light-front decomposition of the momentum $P' = (P'^-, P'^+, P'_\perp)$, where $P'^\pm = P'^0 \pm P'^3$, so that $P'^2 = P'^+P'^- - P'^2_\perp$, and we work in the $q^+ = 0$ frame. The incoming and outgoing mesons have the momentum $P' = p'_1 + p_2$ and $P'' = p''_1 + p_2$, respectively. The quark and antiquark inside the incoming (outgoing) meson have the masses $m_1^{\prime(\prime\prime)}$ and $m_2$, whose momenta are denoted $p_1^{\prime(\prime\prime)}$ and $p_2$, respectively. These momenta can be expressed in terms of the internal variables $(x_i, p'_\perp)$ as

$$p'^+_{1,2} = x_{1,2} P'^+ , \quad p'_{1,2\perp} = x_{1,2} P'_\perp \pm p'_\perp , \tag{9}$$

with $x_1 + x_2 = 1$. Using these internal variables, we can define some other useful quantities for the incoming meson:

$$M_0'^2 = (e'_1 + e_2)^2 = \frac{p'^2_\perp + m'^2_1}{x_1} + \frac{p'^2_\perp + m_2^2}{x_2} ,$$
$$\widetilde{M}'_0 = \sqrt{M_0'^2 - (m'_1 - m_2)^2} ,$$
$$e_i^{(\prime)} = \sqrt{m_i^{(\prime)2} + p'^2_\perp + p'^2_z} , \quad p'_z = \frac{x_2 M'_0}{2} - \frac{m_2^2 + p'^2_\perp}{2x_2 M'_0} . \tag{10}$$

$e_i^{(\prime)}$ can be interpreted as the energy of the quark or the antiquark, and $M'_0$ can be viewed as the kinematic invariant mass of the meson system. To calculate the amplitude for the transition form factor, we need the following Feynman rules for the meson–quark–antiquark vertices $(\mathrm{i}\Gamma'_M)^2$:

$$\mathrm{i}\Gamma'_P = H'_P \gamma_5 , \tag{11}$$
$$\mathrm{i}\Gamma'_V = \mathrm{i}H'_V \left[\gamma_\mu - \frac{1}{W'_V}(p'_1 - p_2)_\mu\right] , \tag{12}$$
$$\mathrm{i}\Gamma'_A = -H'_A \left[\gamma_\mu + \frac{1}{W'_A}(p'_1 - p_2)_\mu\right]\gamma_5 . \tag{13}$$

Here and in the following, we use the subscript A to denote the axial-vector with the quantum numbers $J^{PC} = 1^{++}$. For the outgoing meson, we should use $\mathrm{i}(\gamma_0 \Gamma_M'^\dagger \gamma_0)$ for the corresponding vertices.

In the conventional light-front quark model, the constituent quarks are required to be on mass shell, and the physical quantities can be extracted from the plus component of the corresponding current matrix elements. However, this framework suffers from the problem of noncovariance and missing zero-mode contributions. To solve this problem, Jaus proposed the covariant light-front approach, which can deal with the zero-mode contributions systematically [24]. Decay constants and form factors can be calculated in terms of Feynman momentum loop integrals that are manifestly covariant. In this framework, the lowest order contribution to the form factor is depicted in Fig. 1. For the $P \to V$ transition, the decay amplitudes are

$$\mathcal{B}_\mu^{PV} = -\mathrm{i}^3 \frac{N_c}{(2\pi)^4} \int \mathrm{d}^4 p'_1 \frac{H'_P(\mathrm{i}H''_V)}{N'_1 N''_1 N_2} S_{\mu\nu}^{PV} \varepsilon''^{*\nu} , \tag{14}$$

where $N_1^{\prime(\prime\prime)} = p_1^{\prime(\prime\prime)2} - m_1^{\prime(\prime\prime)2} + \mathrm{i}\epsilon$, $N_2 = p_2^2 - m_2^2 + \mathrm{i}\epsilon$ and

$$\begin{aligned} S_{\mu\nu}^{PV} &= \left(S_V^{PV} - S_A^{PV}\right)_{\mu\nu} \\ &= \mathrm{Tr}\Big[\Big(\gamma_\nu - \frac{1}{W''_V}(p''_1 - p_2)_\nu\Big) \\ &\qquad \times (\not p''_1 + m''_1)(\gamma_\mu - \gamma_\mu\gamma_5)(\not p'_1 + m'_1)\gamma_5(-\not p_2 + m_2)\Big] \\ &= -2\mathrm{i}\epsilon_{\mu\nu\alpha\beta}\big\{p_1'^\alpha P^\beta (m''_1 - m'_1) \\ &\quad + p_1'^\alpha q^\beta (m''_1 + m'_1 - 2m_2) + q^\alpha P^\beta m'_1\big\} \\ &\quad + \frac{1}{W''_V}(4p'_{1\nu} - 3q_\nu - P_\nu)\mathrm{i}\epsilon_{\mu\alpha\beta\rho} p_1'^\alpha q^\beta P^\rho \\ &\quad + 2g_{\mu\nu}\big\{m_2\big(q^2 - N'_1 - N''_1 - m_1'^2 - m_1''^2\big) \\ &\quad - m'_1\big(M''^2 - N''_1 - N_2 - m_1''^2 - m_2^2\big) \\ &\quad - m''_1\big(M'^2 - N'_1 - N_2 - m_1'^2 - m_2^2\big) - 2m'_1 m''_1 m_2\big\} \\ &\quad + 8p'_{1\mu}p'_{1\nu}(m_2 - m'_1) - 2(P_\mu q_\nu + q_\mu P_\nu + 2q_\mu q_\nu)m'_1 \\ &\quad + 2p'_{1\mu}P_\nu(m'_1 - m''_1) + 2p'_{1\mu}q_\nu(3m'_1 - m''_1 - 2m_2) \\ &\quad + 2P_\mu p'_{1\nu}(m'_1 + m''_1) + 2q_\mu p'_{1\nu}(3m'_1 + m''_1 - 2m_2) \\ &\quad + \frac{1}{2W''_V}(4p'_{1\nu} - 3q_\nu - P_\nu)\big\{2p'_{1\mu}\big[M'^2 + M''^2 - q^2 \\ &\quad - 2N_2 + 2(m'_1 - m_2)(m''_1 + m_2)\big] \\ &\quad + q_\mu\big[q^2 - 2M'^2 + N'_1 - N''_1 + 2N_2 \\ &\quad - (m_1 + m''_1)^2 + 2(m'_1 - m_2)^2\big] \\ &\quad + P_\mu\big[q^2 - N'_1 - N''_1 - (m'_1 + m''_1)^2\big]\big\} . \end{aligned} \tag{15}$$

In practice, we use the light-front decomposition of the loop momentum and have to perform the integration over the minus component using the contour method, as the covariant vertex functions cannot be determined by solving the bound state equation. If the covariant vertex functions are not singular when performing the integration, the tran-

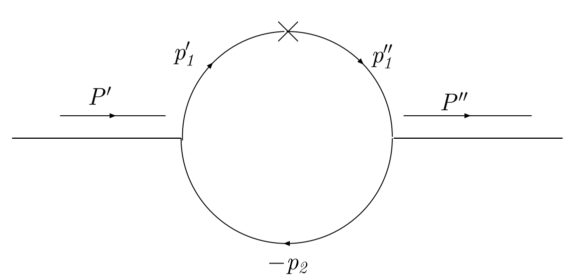


**Fig. 1.** Feynman diagram for $B_c \to J/\psi(X(3872))$ decay amplitudes. The X in the diagram denotes the $V-A$ transition vertex while the meson–quark–antiquark vertices are given in the text

[2] We use a phase by $-\mathrm{i}$ different from [25, 26] for the incoming axial-vector vertex.

sition amplitude will pick up the singularities in the antiquark propagator. The integration leads to

$$\begin{aligned} N_1^{\prime(\prime\prime)} &\to \hat{N}_1^{\prime(\prime\prime)} = x_1\left(M^{\prime(\prime\prime)2} - M_0^{\prime(\prime\prime)2}\right), \\ H_M^{\prime(\prime\prime)} &\to h_M^{\prime(\prime\prime)}, \\ W_M^{\prime\prime} &\to w_M^{\prime\prime}, \end{aligned}$$

$$\int \frac{\mathrm{d}^4 p_1'}{N_1' N_1'' N_2} H_P' H_V'' S \to -\mathrm{i}\pi \int \frac{\mathrm{d}x_2 \mathrm{d}^2 p_\perp'}{x_2 \hat{N}_1' \hat{N}_1''} h_P' h_V'' \hat{S}, \quad (16)$$

where

$$M_0^{\prime\prime 2} = \frac{p_\perp^{\prime\prime 2} + m_1^{\prime\prime 2}}{x_1} + \frac{p_\perp^{\prime\prime 2} + m_2^2}{x_2}, \quad (17)$$

with $p_\perp'' = p_\perp' - x_2 q_\perp$. The explicit forms of $h_M'$ and $w_M'$ for the pseudoscalar, vector and axial-vector $1^{++}$ are given by [25, 26]

$$\begin{aligned} h_P' = h_V' &= \left(M'^2 - M_0'^2\right)\sqrt{\frac{x_1 x_2}{N_c}} \frac{1}{\sqrt{2}\widetilde{M}_0'} \varphi', \\ h_A' &= \left(M'^2 - M_0'^2\right)\sqrt{\frac{x_1 x_2}{N_c}} \frac{1}{\sqrt{2}\widetilde{M}_0'} \frac{\widetilde{M}_0'^2}{2\sqrt{2}M_0'} \varphi_p', \quad (18) \end{aligned}$$

$$w_V' = M_0' + m_1' + m_2, \quad w_A' = \frac{\widetilde{M}_0'^2}{m_1' - m_2}, \quad (19)$$

where $\varphi'$ and $\varphi_p'$ are the phenomenological light-front momentum distribution amplitudes for $s$-wave and $p$-wave mesons, respectively. After this integration, the conventional light-front model is recovered, but manifestly the covariance is lost, as it receives additional spurious contributions proportional to the lightlike four-vector $\tilde{\omega} = (\tilde{\omega}^-, \tilde{\omega}^+, \tilde{\omega}_\perp) = (1, 0, 0_\perp)$. The spurious contributions can be eliminated by including the zero-mode contribution, which amounts to performing the $p^-$ integration in a proper way [24–26].

By using (15)–(19) and the integration rules in [24–26], one arrives at

$$\begin{aligned} g(q^2) &= -\frac{N_c}{16\pi^3} \int \mathrm{d}x_2\, \mathrm{d}^2 p_\perp' \frac{2h_P' h_V''}{x_2 \hat{N}_1' \hat{N}_1''} \left\{ x_2 m_1' + x_1 m_2 \right. \\ &\quad \left. + (m_1' - m_1'') \frac{p_\perp' \cdot q_\perp}{q^2} + \frac{2}{w_V''}\left[p_\perp'^2 + \frac{(p_\perp' \cdot q_\perp)^2}{q^2}\right] \right\}, \\ f(q^2) &= \frac{N_c}{16\pi^3} \int \mathrm{d}x_2\, \mathrm{d}^2 p_\perp' \frac{h_P' h_V''}{x_2 \hat{N}_1' \hat{N}_1''} \\ &\quad \times \left\{ 2x_1(m_2 - m_1')\left(M_0'^2 + M_0''^2\right) - 4x_1 m_1'' M_0'^2 \right. \\ &\quad + 2x_2 m_1' q \cdot P + 2m_2 q^2 - 2x_1 m_2\left(M'^2 + M''^2\right) \\ &\quad + 2(m_1' - m_2)(m_1' + m_1'')^2 \\ &\quad + 8(m_1' - m_2)\left[p_\perp'^2 + \frac{(p_\perp' \cdot q_\perp)^2}{q^2}\right] \\ &\quad + 2(m_1' + m_1'')(q^2 + q \cdot P)\frac{p_\perp' \cdot q_\perp}{q^2} \\ &\quad - 4\frac{q^2 p_\perp'^2 + (p_\perp' \cdot q_\perp)^2}{q^2 w_V''}\left[2x_1\left(M'^2 + M_0'^2\right) - q^2 - q \cdot P \right. \\ &\quad \left.\left. - 2(q^2 + q \cdot P)\frac{p_\perp' \cdot q_\perp}{q^2} - 2(m_1' - m_1'')(m_1' - m_2)\right]\right\}, \end{aligned}$$

$$\begin{aligned} a_+(q^2) &= \frac{N_c}{16\pi^3} \int \mathrm{d}x_2\, \mathrm{d}^2 p_\perp' \frac{2h_P' h_V''}{x_2 \hat{N}_1' \hat{N}_1''} \\ &\quad \times \{(x_1 - x_2)(x_2 m_1' + x_1 m_2) \\ &\quad - [2x_1 m_2 + m_1'' + (x_2 - x_1) m_1'] \frac{p_\perp' \cdot q_\perp}{q^2} \\ &\quad - 2\frac{x_2 q^2 + p_\perp' \cdot q_\perp}{x_2 q^2 w_V''} \\ &\quad \times [p_\perp' \cdot p_\perp'' + (x_1 m_2 + x_2 m_1')(x_1 m_2 - x_2 m_1'')]\}, \end{aligned} \quad (20)$$

while the physical form factors are related to the above functions by

$$\begin{aligned} V^{PV}(q^2) &= -(m_{B_c} + m_{J/\psi}) g(q^2), \\ A_1^{PV}(q^2) &= -\frac{f(q^2)}{m_{B_c} + m_{J/\psi}}, \\ A_2^{PV}(q^2) &= (m_{B_c} + m_{J/\psi}) a_+(q^2). \end{aligned} \quad (21)$$

The extension to $P \to A$ transitions is straightforward:

$$\mathcal{B}_\mu^{PA} = -\mathrm{i}^3 \frac{N_c}{(2\pi)^4} \int \mathrm{d}^4 p_1' \frac{H_P' H_A''}{N_1' N_1'' N_2} S_{\mu\nu}^{PA} \varepsilon''^{*\nu}, \quad (22)$$

where

$$\begin{aligned} S_{\mu\nu}^{PA} &= \left(S_V^{PA} - S_A^{PA}\right)_{\mu\nu} \\ &= \mathrm{Tr}\left[\left(\gamma_\nu - \frac{1}{W_A''}(p_1'' - p_2)_\nu\right)\gamma_5(\not{p}_1'' + m_1'') \right. \\ &\quad \left. \times (\gamma_\mu - \gamma_\mu\gamma_5)(\not{p}_1' + m_1')\gamma_5(-\not{p}_2 + m_2)\right] \\ &= \mathrm{Tr}\left[\left(\gamma_\nu - \frac{1}{W_A''}(p_1'' - p_2)_\nu\right)(\not{p}_1'' - m_1'') \right. \\ &\quad \left. \times (\gamma_\mu\gamma_5 - \gamma_\mu)(\not{p}_1' + m_1')\gamma_5(-\not{p}_2 + m_2)\right]. \end{aligned} \quad (23)$$

By comparing (15) and (23), we have $S_{V(A)}^{PA} = S_{A(V)}^{PV}$ with the replacement $m_1'' \to -m_1''$, $W_V'' \to W_A''$, except for a phase i. Consequently, the form factors of $B \to A$ can be related to the $B \to V$ form factors through

$$\begin{aligned} &\ell^A(q^2) = f(q^2) \text{ with} \\ &\quad m_1'' \to -m_1'', \quad h_V'' \to h_A'', \quad w_V'' \to w_A'', \\ &q^A(q^2) = g(q^2) \text{ with} \\ &\quad m_1'' \to -m_1'', \quad h_V'' \to h_A'', \quad w_V'' \to w_A'', \\ &c_+^A(q^2) = a_+(q^2) \text{ with} \\ &\quad m_1'' \to -m_1'', \quad h_V'' \to h_A'', \quad w_V'' \to w_A'', \end{aligned} \quad (24)$$

where we should be cautious that the replacement of $m_1'' \to -m_1''$ cannot be applied to $m_1''$ in $w''$ and $h''$. We have

$$\begin{aligned} A^{PA}(q^2) &= -(m_{B_c} - m_X) q(q^2), \\ V_1^{PA}(q^2) &= -\frac{\ell(q^2)}{m_{B_c} - m_X}, \\ V_2^{PA}(q^2) &= (m_{B_c} - m_X) c_+(q^2). \end{aligned} \quad (25)$$

In the above expressions for the form factors, there are many terms containing $(p'_\perp \cdot q_\perp)/q^2$ in the integrand. These terms can make non-trivial contributions together with $h''_M/\hat{N}''_1$. In the calculation, we make a Taylor expansion for $h''_M/\hat{N}''_1$ as follows:

$$\frac{h''_V}{\hat{N}''_1} = \frac{h''_V}{\hat{N}''_1}\bigg|_{p''^2_\perp \to p'^2_\perp} - 2x_2 p'_\perp \cdot q_\perp \left(\frac{\mathrm{d}}{\mathrm{d}p''^2_\perp}\frac{h''_V}{\hat{N}''_1}\right)_{p''^2_\perp \to p'^2_\perp} + \mathcal{O}\left(x_2^2 q^2\right). \tag{26}$$

Then terms containing $p'_\perp \cdot q_\perp$ can be simplified using the following equation:

$$\int \mathrm{d}^2 p'_\perp \frac{(p'_\perp \cdot q_\perp)^2}{q^2} = -\frac{1}{2}\int \mathrm{d}^2 p'_\perp p'^2_\perp . \tag{27}$$

Now it is straightforward to obtain the decay width:

$$\Gamma(B_c^- \to J/\psi\pi^-) = \frac{|G_\mathrm{F} V_{cb} V^*_{ud} a_1 f_\pi m^2_{B_c} A_0^{PV}(0)|^2}{32\pi m_{B_c}} \times \left(1 - r^2_{J/\psi}\right), \tag{28}$$

where $r_{J/\psi} = \frac{m_{J/\psi}}{m_{B_c}}$. For the decays involving $K^-$, the factor $V^*_{ud} f_\pi$ is replaced by $V^*_{us} f_K$, while for $B_c^- \to X\pi^-(K^-)$, $A_0^{PV}(0)$ $(r_{J/\psi})$ is replaced by $V_0^{PA}(0)$ $(r_X)$.

## 3 Numerical results and discussion

To perform the numerical calculations we need to specify the input parameters in the covariant light-front framework. The $\bar{q}q$ meson state is described by the light-front wave function which can be obtained by solving the relativistic Schrödinger equation with a phenomenological potential. But, in fact, except for some special cases, the solution is not obtainable at present. We prefer to employ a phenomenological wave function to describe the hadronic structure. In the present work, we shall use the simple Gaussian-type wave function [33]

$$\varphi' = \varphi'\left(x_2, p'_\perp\right) = 4\left(\frac{\pi}{\beta^2}\right)^{\frac{3}{4}} \sqrt{\frac{\mathrm{d}p'_z}{\mathrm{d}x_2}} \exp\left(-\frac{p'^2_z + p'^2_\perp}{2\beta^2}\right),$$
$$\varphi'_p = \varphi'_p(x_2, p'_\perp) = \sqrt{\frac{2}{\beta^2}}\varphi', \quad \frac{\mathrm{d}p'_z}{\mathrm{d}x_2} = \frac{e'_1 e_2}{x_1 x_2 M'_0}. \tag{29}$$

The input parameters $m_q$ and $\beta$ in the Gaussian-type wave function (29) are shown in Table 1. The constituent quark masses are close to those used in the literature [20–22, 24–28]. The parameter $\beta$, which describes the momentum distribution, is expected to be of order $\Lambda_{\mathrm{QCD}}$. These parameters $\beta$ are fixed by the decay constants, whose analytic expressions in the covariant light-front model are given in [25, 26]. The decay constant $f_{J/\psi}$ can be determined by the leptonic decay width:

$$\Gamma_{ee} \equiv \Gamma(J/\psi \to e^+e^-) = \frac{4\pi\alpha^2_{\mathrm{em}} Q_c^2 f^2_{J/\psi}}{3 m_{J/\psi}}, \tag{30}$$

where $Q_c = 2/3$, denotes the electric charge of the charm quark. Using the measured results for the electronic width of $J/\psi$ [34],

$$\Gamma_{ee} = (5.55 \pm 0.14 \pm 0.02)\,\mathrm{keV}, \tag{31}$$

we obtain $f_{J/\psi} = 0.416 \pm 0.05$ GeV. As for the decay constant for $B_c$ and $X$, we use $f_{B_c} = 0.398^{+0.054}_{-0.055}$ GeV, and $|f_{X(3872)}| = 0.329^{+0.111}_{-0.095}$ GeV. For the light pseudoscalars, we use $f_\pi = 0.132$ GeV and $f_K = 0.16$ GeV. The determined results for $\beta$ are listed in Table 1.

In the factorization approach, the decay amplitude is expressed as a product of the short distance Wilson coefficients, the form factors and the meson decay constant. The latter two are physical values that are scale independent. But the Wilson coefficient $a_1$ of the four-quark operators depends on the factorization scale. This directly leads to the scale dependence of the decay amplitude. But, as we have shown in [35], the numerical value of $a_1$ is not very sensitive to the scale; thus we use $a_1 = 1.1$ in this work. The CKM matrix elements, the lifetime of the $B_c$ meson and the masses of the hadrons are chosen from the Particle Data Group [34]:

$$|V_{cb}| = 0.0416 \pm 0.0006, \quad |V_{ud}| = 0.97377 \pm 0.00027, \quad |V_{us}| = 0.2257 \pm 0.021, \tag{32}$$
$$m_{B_c} = 6.286\ \mathrm{GeV}, \quad \tau_{B_c} = (0.46^{+0.18}_{-0.16}) \times 10^{-12}\,\mathrm{s}, \tag{33}$$
$$m_{J/\psi} = 3.097\ \mathrm{GeV}, \quad m_X = 3.872\ \mathrm{GeV}. \tag{34}$$

The uncertainties in the above CKM matrix elements are small; thus, they induce small errors to the decay width, and we will neglect these uncertainties.

Using the above input, we can calculate the form factors directly. As in [24–26], because of the condition $q^+ = 0$ that we have imposed during the course of calculation, form factors are known only for spacelike momentum transfer $q^2 = -q^2_\perp \le 0$. But only the timelike form factors are relevant for the physical decay processes. It has been proposed in [20, 21] to recast the form factors as explicit functions of $q^2$ in the spacelike region and then analytically extrapolate them to the timelike region. In exclusive non-leptonic decays, only the form factor at maximally recoiling ($q^2 \simeq 0$) is required, and therefore we do not need to discuss the dependence on the momentum transfer here. After the calculation, the results for the $B_c \to J/\psi$ and $B_c \to X$ (assuming

**Table 1.** The input parameters $m_q$ and $\beta$ (in unit of GeV) in the Gaussian-type light-front wave function (29)

| $m_c$ | $m_b$ | $\beta_{B_c}$ | $\beta_{J/\psi}$ | $\beta_X$ |
|---|---|---|---|---|
| 1.4 | 4.4 | $0.870 \pm 0.100$ | $0.631^{+0.06}_{-0.04}$ | $0.720 \pm 0.100$ |

$X$ as a $1^{++}$ state) form factors are

$$\begin{aligned}
&V^{PV}(0) = 0.87^{+0.00+0.01}_{-0.02-0.00}\,,\\
&A_0^{PV}(0) = A_3^{PV}(0) = 0.57^{+0.01+0.00}_{-0.02-0.00}\,,\\
&A_1^{PV}(0) = 0.55^{+0.01+0.00}_{-0.03-0.00}\,, \qquad A_2^{PV}(0) = 0.51^{+0.03+0.00}_{-0.04-0.00}\,,\\
&A^{PA}(0) = 0.36^{+0.02+0.01}_{-0.02-0.03}\,,\\
&V_0^{PA}(0) = V_3^{PA}(0) = 0.18^{+0.01+0.01}_{-0.02-0.02}\,,\\
&V_1^{PA}(0) = 1.15^{+0.03+0.03}_{-0.04-0.06}\,, \qquad V_2^{PA}(0) = 0.13^{+0.02+0.00}_{-0.02-0.01}\,,
\end{aligned} \tag{35}$$

where the first uncertainty is from the decay constant of the $B_c$ meson and the latter is from the charmonium decay constant. In Table 2, we make a comparison of our results with the previous studies. We can see that our results are slightly smaller than the results from the three point sum rule and the quark model, but the light-cone sum rule predictions are quite different from the other ones.

Using the results for the $B_c \to J/\psi$ form factors, we obtain the branching ratio of $B_c^- \to J/\psi\pi^-(K^-)$:

$$\begin{aligned}
\mathrm{BR}(B_c^- \to J/\psi\pi^-) &= \left(2.0^{+0.8+0.0+0.0}_{-0.7-0.1-0.0}\right) \times 10^{-3},\\
\mathrm{BR}(B_c^- \to J/\psi K^-) &= \left(1.6^{+0.6+0.0+0.0}_{-0.6-0.1-0.0}\right) \times 10^{-4},
\end{aligned} \tag{36}$$

where the uncertainties are from the large errors in the lifetime of the $B_c$ meson, the decay constant of the $B_c$ meson and the charmonium decay constants. In the literature, these decays have been subject to extensive study [40–52], and the range of the branching ratios is

$$\begin{aligned}
\mathrm{BR}(B_c^- \to J/\psi\pi^-) &= (0.06\text{–}0.18)\%\,,\\
\mathrm{BR}(B_c^- \to J/\psi K^-) &= (0.005\text{–}0.014)\%\,,
\end{aligned} \tag{37}$$

which are values consistent with ours. Assuming $X$ as a $1^{++}$ state, the branching ratios of $B_c^- \to X(3872)\pi^-(K^-)$ are

$$\begin{aligned}
\mathrm{BR}(B_c^- \to X(3872)\pi^-) &= \left(1.7^{+0.7+0.1+0.4}_{-0.6-0.2-0.4}\right) \times 10^{-4},\\
\mathrm{BR}(B_c^- \to X(3872)K^-) &= \left(1.3^{+0.5+0.1+0.3}_{-0.5-0.2-0.3}\right) \times 10^{-5}.
\end{aligned} \tag{38}$$

These results are one order of magnitude smaller than the branching ratio of $B_c^- \to J/\psi\pi^-$ and $B_c^- \to J/\psi K^-$, respectively. From the decay width formulae in (28), we know that the $B_c \to J/\psi P$ branching ratios are proportional to the form factor $|A^{PV}(0)|^2$, while for $B_c \to X(1^{++})P$, the decay widths are proportional to $|V^{PA}(0)|^2$. The $B_c \to X$ form factor $V^{PA}(0)$ is only 1/3 of $|A^{PV}(0)|$ as shown in (35), which induces the one order of magnitude difference for these two kinds of decay.

For $B_c \to VV$ decays, there are three different polarizations. According to the power counting rule in the standard model [53–55], the longitudinal polarization dominates in the decay processes, while other polarizations suffer from one or two orders of $\Lambda_{\mathrm{QCD}}/m_B$ or $m_c/m_B$ suppressions that arise from the quark helicity flip. It is found that the annihilation diagrams with the operator $O_6$ could violate this power counting rule [56–58]. However, in the $B_c^- \to J/\psi\rho^-(K^{*-})$ decays, there are only emission-type contributions; thus the power counting rule should work well. If we neglect the $m_\rho^2/m_{B_c}^2$ terms in the polarization vector, the formulae for the branching ratios of $B \to VV$ are the same as (28) with the replacement of the decay constant $f_P \to f_V$. Using $f_\rho = 0.209\,\mathrm{GeV}$ and $f_{K^*} = 0.217\,\mathrm{GeV}$, we obtain the corresponding branching ratios:

$$\begin{aligned}
\mathrm{BR}(B_c^- \to J/\psi\rho^-) &= \left(5.0^{+2.0+0.1+0.1}_{-1.7-0.0-0.0}\right) \times 10^{-3},\\
\mathrm{BR}(B_c^- \to J/\psi K^{*-}) &= \left(2.9^{+1.1+0.0+0.0}_{-1.0-0.0-0.0}\right) \times 10^{-4},\\
\mathrm{BR}(B_c^- \to X(3872)\rho^-) &= \left(4.1^{+1.6+0.3+0.1}_{-1.4-0.1-0.1}\right) \times 10^{-4},\\
\mathrm{BR}(B_c^- \to X(3872)K^{*-}) &= \left(2.4^{+0.9+0.2+0.5}_{-0.8-0.3-0.5}\right) \times 10^{-5}.
\end{aligned} \tag{39}$$

Our above calculation is based on the $1^{++}$ charmonium description for $X$. The charmonium states with other quantum numbers can also be studied similarly in this approach. If the quantum numbers of $X(3872)$ are changed to $1^{--}$, the large form factor $A_0^{B_c \to X}$ can enhance the production rates dramatically:

$$\begin{aligned}
\mathrm{BR}(B_c^- \to X(3872)\pi^-) &= \left(1.4^{+0.6+0.0+0.4}_{-0.5-0.0-0.5}\right) \times 10^{-3},\\
\mathrm{BR}(B_c^- \to X(3872)K^-) &= \left(1.1^{+0.4+0.0+0.3}_{-0.4-0.0-0.4}\right) \times 10^{-4},\\
\mathrm{BR}(B_c^- \to X(3872)\rho^-) &= \left(3.5^{+1.4+0.0+1.1}_{-1.2-0.2-1.2}\right) \times 10^{-3},\\
\mathrm{BR}(B_c^- \to X(3872)K^{*-}) &= \left(2.0^{+0.8+0.0+0.6}_{-0.7-0.1-0.7}\right) \times 10^{-4}.
\end{aligned} \tag{40}$$

Comparing the above equations with (38) and (39), we see that the production rates are enhanced by about one order of magnitude. The large branching ratios and the large difference between the different quantum numbers of state $X$ can easily be used at the LHCb experiment to test the charmonium description for $X$.

**Table 2.** The values of the form factors of $B_c \to J/\psi$ at $q^2 = 0$ in comparison with the estimates in the three point sum rule (3PSR) (with the Coloumb corrections included) [36, 37], in the quark model (QM) [38] and the light-cone sum rule (LCSR) approach [39]

| | $A_1$ | $A_2$ | $V$ |
|---|---|---|---|
| 3PSR [36, 37] | 0.63 | 0.69 | 1.03 |
| QM [38] | 0.68 | 0.66 | 0.96 |
| LCSR [39] | 0.75 | 1.69 | 1.69 |
| This work | $0.55^{+0.01+0.00}_{-0.03-0.00}$ | $0.51^{+0.03+0.00}_{-0.04-0.00}$ | $0.87^{+0.00+0.01}_{-0.02-0.00}$ |

## 4 Conclusion

In the covariant light-front quark model, we study the form factors of the $B_c \to J/\psi$ and $B_c \to X$ transitions at maximum recoiling. The factorization of exclusive processes $B_c^- \to J/\psi\pi^-(K^-)$ and $B_c^- \to X(3872)\pi^-(K^-)$ can be proved to all orders of the strong coupling constant just as in the proof for $\bar{B}^0 \to D^+\pi^-$ and $B^- \to D^0\pi^-$. Therefore, the decay widths of these decays can simply be calculated in the naive factorization approach utilizing the form factors. Our results for the branching ratio of $B_c \to J/\psi\pi^-(K^-)$ are consistent with the previous studies considering the uncertainties. The study of these exclusive processes may greatly improve our understanding of the $B_c$ meson exclusive hadronic decays, and the corresponding theory describing them as well. In our calculation, identifying $X(3872)$ as a $1^{++}$ charmonium state, we obtain $\mathrm{BR}(B_c^- \to X(3872)\pi^-) = (1.7^{+0.7+0.1+0.4}_{-0.6-0.2-0.4}) \times 10^{-4}$ and $\mathrm{BR}(B_c^- \to X(3872)K^-) = (1.3^{+0.5+0.1+0.3}_{-0.5-0.2-0.3}) \times 10^{-5}$. Assuming $X$ to be a $1^{--}$ state, the branching ratios are one order of magnitude larger. This large difference can easily be used by the LHCb experiment to test the different charmonium descriptions for $X$.

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